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Letter to the Editor

Comments on "Constructal design of a thermoelectric device", IJHMT 49 (2006) 1420–1429

A recent publication applies the finite time thermodynamics and constructal theory to study the distribution of Joule heat going into the hot and cold sides of a TEG and its consequences on optimal allocation of heat exchanger inventory [1]. Nonetheless, the heat transfer model used is incomplete and hence the main results and conclusions of the paper concerning the Joule heat are questionable. Based on a more complete formulation, we shall comment one conclusion relevant to TEG constructal design methodology published in Ref. [1] in this communication. The conclusion involved is the causal relation between the conductance allocation of external heat exchangers and the Joule heat affecting the two junctions in Ref. [1]. In their words, "the necessary and sufficient condition for equipartition of Joulean heat produced is the equipartition of conductance allocations between high temperature and low temperature heat sources".

The basic element of a real TEG module is a pair of thermo-legs such as shown in Fig. 1 of [1]. Before the discussion of the heat transfer procedure of the single-couple TEG, it is helpful to get a clear idea about the temperature and heat flow distribution of either the n-type semiconductor leg or the p-type semiconductor leg, a generic conductive bar both thermally and electrically, as shown in Fig. 2 of [1]. According to the thermal arrangements of Fig. 4 of [1], where Thomson effect is neglected, the bar is sandwiched between two thermally conducting but electrically insulating heat exchangers mounted above and below (K_H and K_L) (The symbols used here shall follow the same meanings as in Ref. [1] unless otherwise stated.). Consequently the external source of irreversibility is incorporated in the model, and the hot and cold sides of the bar are open to the hot source T_H and cold source T_L through K_H and K_L, respectively.

The first issue that needs to be considered is the temperature and heat flow distribution in the bar when $T_{HC} = T_{LC}$. If the bar is adiabatic from the surroundings except the heat flows at the two ends, as is assumed by [1], it must be isothermal with the temperature boundary condition imposed when the current is absent. The temperature distribution in the bar will be altered with a current flowing in it. Roughly speaking, the temperature of bar will increase. The reason of the temperature increase is that the Joule heat generated in the bar must flow towards both ends by conduction, and as long as there is heat conduction, there must be an accompanying temperature gradient. According to the assumptions of Ref. [1] that the thermal conductivity and electrical resistivity of the material are constant, neglecting Thomson effect, the 1-D differential equation governing the heat flow Q of the infinitesimal element with a length of dx in the bar under steady state, is:

$$dQ = Q(x + dx) - Q(x) = I^2 \rho \frac{dx}{A} \tag{1}$$

where A is uniform cross-sectional area of the bar. Here the plus direction of Q is defined as from T_{HC} to T_{LC}, namely, from $x = 0$ to $x = L$. It is clear that Eq. (1) applies to both the plus heat flow and the minus heat flow. Hence the heat flow of the two sides can be obtained as the following when $T_{HC} = T_{LC}$,

$$Q(0) = K(T_{HC} - T_{LC}) - \frac{I^2 R}{2} = -\frac{I^2 R}{2} \tag{2}$$

$$Q(L) = K(T_{HC} - T_{LC}) + \frac{I^2 R}{2} = \frac{I^2 R}{2} \tag{3}$$

by solving (1) and Fourier's heat conduction equation.

$$Q(x) = -kA \frac{dT(x)}{dx} \tag{4}$$

The sign in front of the Joule heat term of Eqs. (2) and (3) represents the heat flow direction. So these two equations show that, when $T_{HC} = T_{LC}$, exactly half of the Joule heat arrives at both hot and cold sides, and it is natural to imagine that the maximum temperature should pass through the geometrical center of the bar, namely, in the place $x = 0.5L$. This postulate can be mathematically proven from the temperature distribution function of the bar,

$$T(x) = -\frac{I^2 \rho}{2kA^2} x^2 + \left(\frac{I^2 \rho L}{2kA^2} - \frac{(T_{HC} - T_{LC})}{L} \right) x + T_{HC} \tag{5}$$

which is obtained also by solving the differential equation (1) and Fourier's heat conduction equation (4). The derivative of the parabolic temperature distribution function (5) is

$$\frac{dT(x)}{dx} = -\frac{I^2 \rho}{kA^2} x + \frac{I^2 \rho L}{2kA^2} - \frac{(T_{HC} - T_{LC})}{L} \tag{6}$$

Given the temperature boundary condition $T_{HC} = T_{LC}$, the solution of the location of maximum temperature in the bar, obtained by setting Eq. (6) equal to zero, reads as

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$$x = \frac{L}{2} - \frac{k(T_{HC} - T_{LC})A^2}{I^2 \rho L} = \frac{L}{2} \quad (7)$$

The Peltier heat exchange due to current passing dissimilar materials needs not necessarily to be considered in the foregoing 1-D analysis of the bar. When it is temporarily neglected at the current stage, a heat flow diagram, which offers a physical picture for this situation, is shown in Fig. 1a. Based upon the heat transfer direction at the two boundary points, $x = 0$ and $x = L$, one can easily draw the conclusion that the temperatures of the both heat sources T_H and T_L must be less than T_{HC} and T_{LC}, respectively; i.e., $T_H < T_{HC}$ and $T_L < T_{LC}$. If we assume an equipartition of the heat exchanger conductance, i.e., $K_H = K_L$, then the temperatures of the two thermal reservoirs are equal, that is, $T_H = T_L$.

The second question that must be considered is: if T_{HC} is increased while T_{LC} is kept at the original value, what will happen after the system has stabilized? We limit the initial change amount of the new temperature boundary condition to be tiny in order to allow another possible situation, i.e., $T_{HC} > T_{LC}$, but $T_{HC} \approx T_{LC}$. Due to the fact that the increase of T_{HC} is limited, the directions of the heat flow of the two sides are not changed in terms of Eqs. (2) and (3). These two equations also clearly disclose that, when $T_{HC} > T_{LC}$, the absolute value of the heat flow of the hot

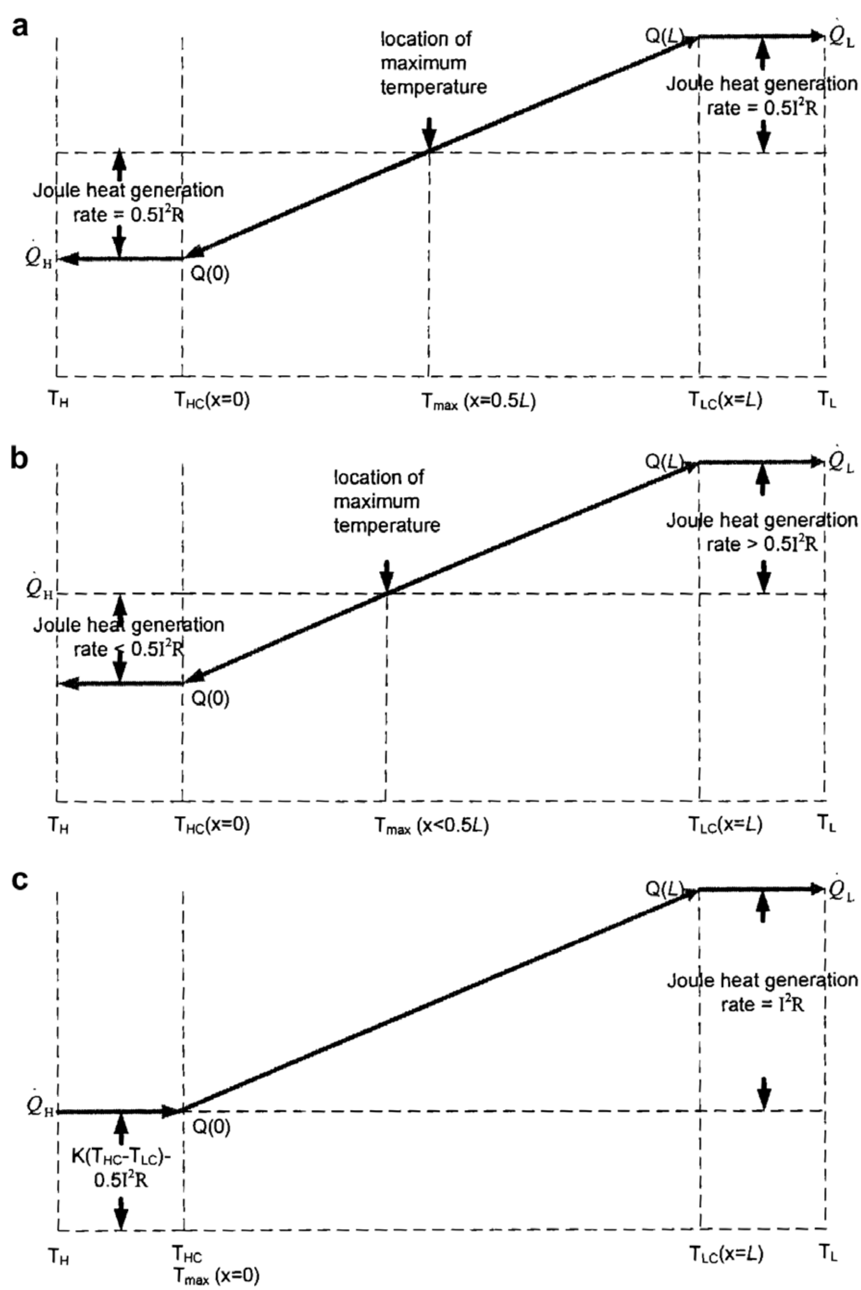

Fig. 1. Heat flow diagram of the conductive bar. (a) $T_{HC} = T_{LC}$ (b) $T_{HC} > T_{LC}$, but $T_{HC} \approx T_{LC}$ (c) $T_{HC} > T_{LC}$, and $K\Delta T \geqslant 0.5I^2R$.

side $Q(0)$ is less than half of the Joule heat, whilst the absolute value of the heat flow of the cold side $Q(L)$ is more than half of the Joule heat. Regarding the location of maximum temperature in the bar, one can deduce that it should move towards the hot side from the center, based on Eq. (7), with the new temperature boundary condition $T_{HC} > T_{LC}$, that is, somewhere at $x < 0.5L$. The heat flow diagram for this case is shown in Fig. 1b. The relations $T_H < T_{HC}$ and $T_L < T_{LC}$ still hold, but T_H must be higher than T_L in case that K_H is still equal to K_L.

Provided that T_{HC} is increased to the extent that $K(T_{HC} - T_{LC})$ is equal to half of the Joule heat $0.5I^2R$, in terms of Eqs. (2) and (3), the heat flow of the hot side $Q(0)$ will be decreased to zero and the heat flow of the cold side $Q(L)$ will be increased to I^2R. So under this condition, all the Joule heat generated in the bar flows to the cold side. Since $Q(0) = 0$, T_H must be equal to T_{HC} in terms of energy conservation. If T_{HC} is further increased, the sign in front of the Joule heat term in Eq. (2) will be changed from minus to plus, which means that the direction of the heat flow $Q(0)$ is changed. Unlike those cases in Fig. 1a and b, now the energy in the heat flow $Q(0)$ is from the heat source rather than the Joule heat. In other words, the heat flow $\dot{Q}_H$ from the heat source enters the bar by conduction, flows to the cold side, and, together with the Joule heat I^2R, constitutes the heat flow of the cold side $Q(L)$ and the heat flow $\dot{Q}_L$ going to the cold source. These two cases are summarized in Fig. 1c.

Noting the fact that the temperature boundary condition T_{HC} and T_{LC} can be changed by changing the external conditions, by changing either the heat and cold sources temperature T_H and T_L or the heat exchanger conductance K_H and K_L, the effect of K_H and K_L on the Joule heat transfer of the bar is at once clear: K_H and K_L are involved in determining the temperature boundary condition T_{HC} and T_{LC}, which consequentially determines the proportion of the Joule heat exactly flowing into the hot and cold end. On the other hand, they have nothing to do with the proportion of the Joule heat affecting the heat flow at the hot and cold ends. Eqs. (2) and (3) are rigorous results of a heat transfer analysis, and coefficient 0.5 in front of the term I^2R is always valid if only the material properties are assumed constant.

The 1-D analysis results obtained so far apply directly to both the p-type and the n-type semiconductor legs, and enables us to proceed to the heat transfer issue of the single-couple TEG, in which the two legs are connected electrically in series by metal strips and a load R_l as is shown in Fig. 1 of [1]. Q is redefined as the heat flow across the whole TEG, the sum of heat flows through the two legs. Accordingly, the definitions of K and R are also altered from thermal conductance and electrical resistance of the bar to those of the TEG. Thus Eqs. (2) and (3) still hold.

The heat flow of a TEG, when operating in usual mode, is similar to the case shown in Fig. 1c, except that there is Peltier heat absorbed at the hot side and evolved at the cold side of the device. The Peltier heat is in reality one part of the heat flow boundary condition. The other part consists of the input $\dot{Q}_H$ and the output $\dot{Q}_L$. There is no doubt that the incorporation of the separate components into the boundary condition is appropriate. Hence, if the two Peltier terms and the thermal conductance of the external heat exchangers are incorporated, we have

$$\dot{Q}_H = K_H(T_H - T_{HC}) = \alpha T_{HC} I + K(T_{HC} - T_{LC}) - \frac{I^2R}{2} \quad (8)$$

$$\dot{Q}_L = K_L(T_{LC} - T_L) = \alpha T_{LC} I + K(T_{HC} - T_{LC}) + \frac{I^2R}{2} \quad (9)$$

where α is the difference of Seebeck coefficients of the p- and n-type materials. The only difference between Eqs. (8), (9) and Eqs. (26–27) of Ref. [1] is the coefficient in front of the Joule term I^2R. The authors of Ref. [1] introduce two new variables F_H and F_L to represent the fraction of Joule heat entering into the hot and cold junction instead of the coefficient 0.5. By including their Eq. (28) ($F_H + F_L = 1$), they believe that they have three equations but four variables, T_{HC}, T_{LC}, F_H and F_L. Hence, there can be a single degree of freedom and they choose F_H to be that one. After a series of manipulations, they present a compilation, in their Table 1, for the fraction of Joule heat distribution for different combinations of conductance allocations of K_H and K_L.

As we have proven, however, the values of K_H and K_L do not have any effect on the fraction of Joule heat affecting the hot and cold junctions. Different conductance allocations can give different temperature boundary conditions, which can in turn lead to different fractions of the Joule heat exactly flowing into the hot and cold junctions. But the coefficient 0.5 in front of I^2R is the result of a rigorous derivation and can never be changed as long as that the material properties are assumed constant.

Actually, the equation system used in Ref. [1] is an incomplete one. The authors ignore the important relation to reflect the coupled behavior between electrical field and thermal field of a TEG.

$$I = \frac{\alpha(T_{HC} - T_{LC})}{R + R_l} \quad (10)$$

With this constraint equation, the correct statement of the problem should be: the system of Eqs. (8)–(10) has three unknown variables T_{HC}, T_{LC} and I rendering a unique solution, which represents an operating state of the TEG. Obviously, there is not any single degree of freedom rendered. The equipartition of Joule heat does not demand the equipartition of conductance allocations of the high temperature and low temperature heat exchangers.

In terms of energy conservation, Eq. (28) of Ref. [1] is always correct, even for the case that the thermal and electrical material properties are not constant. If the material properties are temperature dependent, or inhomogeneous elements are used, the Joule heat affecting the hot and cold junctions will not be exactly one-half any longer because the heat flow change in each leg is not linear any longer. But one still cannot say that there will be 4 equations for 5 unknowns, and hence a single degree of freedom. In the former case, the parameters α, R, and K are dependent on the inside heat flow and temperature distribution of a TEG. They become variables as well, which can describe the TEG state, just like T_{HC}, T_{LC}, I, and the proportion of Joule heat affecting the both sides. Such a treatment is difficult to be tackled analytically and probably can only be done numerically. Some attempts to find the solution by numerical methods have been published [2,3].

Therefore, it turns out that the major result of Ref. [1] about the relation between the conductance allocation and the Joule heat is not correct. The mathematical efforts of Ref. [1] to seek the so-called possible set of solutions for the assumed unknown variable F_H or F_L, and especially the contents in the Table 1, do not make any sense.

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